## DUAL DIG

## LEVEL II

2011

1. Find the missing entry in the following infinite sequence: 1001, 100, $\qquad$ $, 14,13,12,11,10,9,9,9, \ldots$
2. If the sum of two numbers is 1 and their product is 1 , then the sum of their cubes is:
3. Find the sum of the coefficients of all the terms after $(3 x+2 y)^{5}$ is expanded.
4. Calculate $\sum_{k=0}^{10} 3^{k}$
5. Suppose that $\ln 7=a$ and $\ln 11=b$. Write the following logarithm in terms of a and b: $\ln \sqrt[5]{77}$
6. Express -3i in polar form.
7. Find all the vertical asymptotes of the function: $\mathrm{f}(\mathrm{x})=\frac{-x^{3}-2 x^{2}+3 x}{2 x^{3}-x^{2}-25 x-12}$
8. Runners A, B, C, and D have chance of $0.3,0.2,0.1$, and 0.4 respectively of winning a race. If A drops out of the race, what is the probability that B wins the race?
9. Solve: $x^{4}-3 x^{3}-20 x^{2}-24 x-8=0$
10. Determine all asymptotes of the function $\mathrm{f}(\mathrm{x})=\frac{3 x^{2}-x-2}{x-1}$
11. A survey is taken on what kind of sports do students like. Each student checks baseball, football, or basketball as a sport they like. More than one pick is permitted. The results are:

| Football | Basketball | Baseball | Football <br> $\mathbf{\&}$ <br> Basketball | Football <br> $\mathbf{8}$ <br> Baseball | Basketball <br> $\mathbf{8}$ <br> Baseball | All three |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 42 | 34 | 27 | 15 | 12 | 10 | 7 |

How many people completed the survey?
12. Water is poured into a conical paper cup at the rate of $2 / 3$ cubic inches per second. If the cup is 6 inches tall and the top of the cup has a radius of 2 inches, how fast does the water level rise when the water is 4 inches deep?
13. At noon a sailboat is 20 km south of a freighter. The sailboat is traveling east at 20 km per hour, and the freighter is traveling south at 40 km per hour. If visibility is 10 km , could the people on the two ships ever see each other?
14. If $f(x)=\log \left(\frac{1+x}{1-x}\right)$ for $-1<\mathrm{x}<1$, then define $f\left(\frac{3 x+x^{3}}{1+3 x^{2}}\right)$ in terms of $f(x)$ :
15. The statue of Zeus at Olympia in Greece is one of the Seven Wonders of the World. It is made of gold and ivory. The ivory was found to have lost $35 \%$ of its carbon-14. Determine the age of the statue to the nearest year (note: the radioactive element carbon-14 has a half-life of 5750 years).
16. The area of a circle inscribed in a regular hexagon is $100 \pi$. Determine the area of the hexagon.
17. Decompose the following fraction into partial fractions: $\frac{7 x-1}{6 x^{2}-5 x+1}$
18. If two poles $20^{\prime \prime}$ and $80^{\prime \prime}$ high are $100^{\prime \prime}$ apart, then the height of the intersection of the lines joining the top of each pole to the foot of the opposite pole is:
19. Find real numbers x and y such that $3 x+2 i y-i x+5 y=7+5 i$
20. In how many ways can eight people be seated at a round table?

